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Disturbed state concept for partially saturated soils

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ABSTRACT: A general elasto-plastic model for partially saturated soils is proposed, based on the disturbed state concept (DSC). This model is unified and simplified, includes both saturated and unsaturated states and is able to capture some typical features of partially saturated soils. The saturated state can be considered as a special case. Experimental results on a remolded sandy silt are used to calibrate the model.

1. INTRODUCTION

Geological materials such as soils are often partially saturated. However, the majority of theories and models concern fully saturated soils. With the importance of topics such as geo-environmental engineering, dams, tunnels and landslides, considerable attention is now given to the testing and modeling of partially saturated soils.

The Soil Mechanics Laboratory of Lausanne has been active for many years on this subject, and recently focusing on an unsaturated sandy silt, which involves rheological characterization by extensive experimental tests (Laloui *et al.*, 1995) and numerical modeling (Geiser *et al.*, 1997).

The first part of this paper presents a constitutive model for unsaturated soils based on the hierarchical single surface law (HISS models from Desai and coworkers (see Desai, 1994)). The HISS-model initially developed for saturated soils is extended to partially saturated soils by taking into account the evolution of material parameters with the suction. The second part of this paper presents an extension of this approach to allow for the stress softening behavior typically observed in case of low degree of saturation and low net mean pressure (see Cui *et al.*, 1996 & Laloui *et al.*, 1997). The disturbed state concept (DSC) is used for this purpose. It includes the HISS plasticity model as a special case and allows for microcracking, damage and softening, and stiffening (Desai, 1995). The theory of the DSC presented here, for partially saturated soils, was developped by Desai, and is described in details by Desai *et al.* (1996).

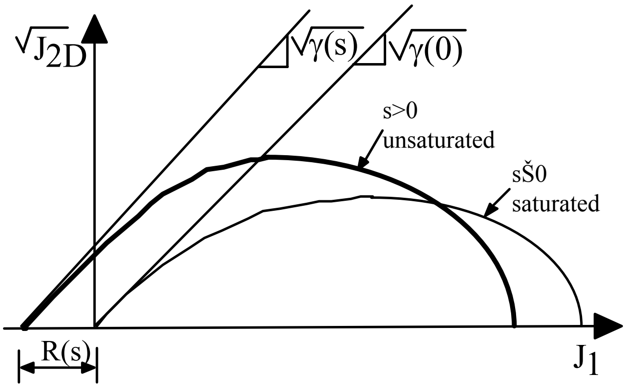

Figure 1: Yield function F

2. MODIFIED $\delta_{1\text{-}m}$

To represent the unsaturated behavior of a porous medium, the non-associative elastoplastic hierarchical single surface model HISS-δ_1, developed initially for saturated soils (Desai, 1994), is modified.

In the following the suction s will be defined as the excess of pore air-pressure u_a to water-pressure u_w(compression positive):

$$s = u_a - u_w \tag{1}$$

The modified HISS-δ1 includes the suction as a state parameter governing the evolution of the yield surface. The expanding yield surface F is defined as:

$$F \equiv J^*_{2D} - \left[-\alpha(J^*_1)^n + \gamma(J^*_1)^2\right](1 - \beta\,\bar{S}_r)^{-0.5} \tag{2}$$

where material parameters are explicit functions of the suction. Figure 1 illustrates the effect of the suction on the shape of F in the $J_1 - \sqrt{J_{2D}}$ plane.

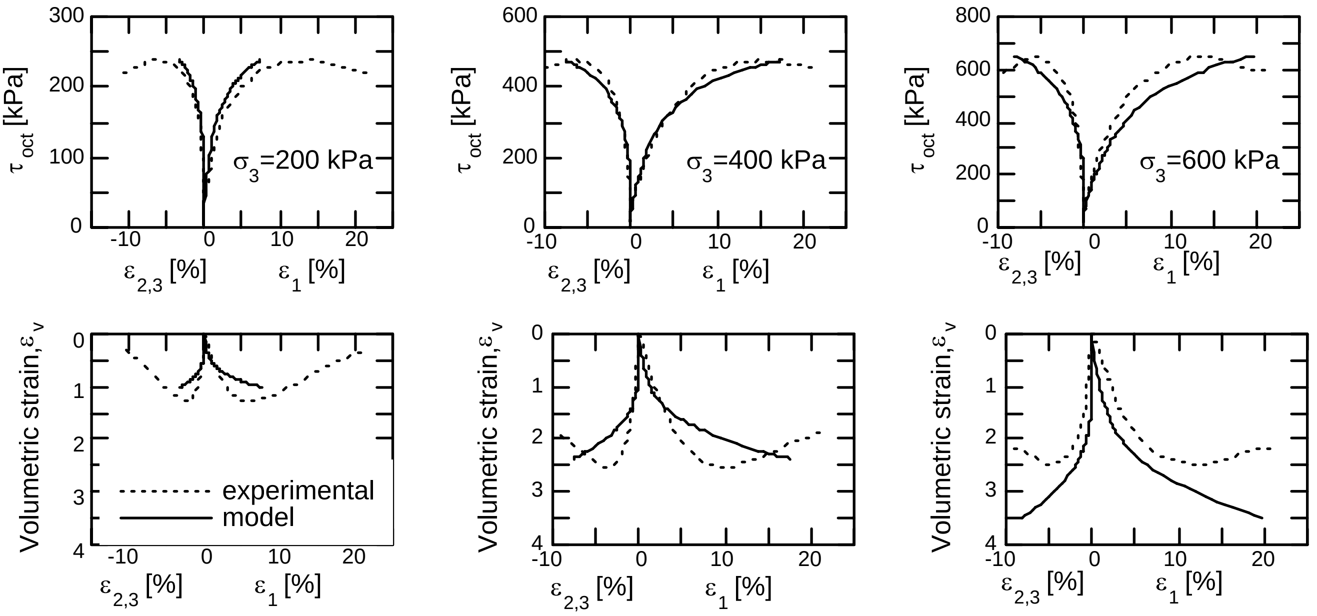

Figure 2: Comparison between experimental results and numerical predictions with the δ_1-model, at three different lateral pressures: σ_3 = 200 kPa, σ_3 = 400 kPa and σ_3 = 600 kPa.

Expressions in Eq. (2) are the following:

$J^*_{2D} = J_{2D} / p_a^2$, with J_{2D} is the second invariant of the deviatoric stress tensor, t_{ij};

$J^*_1 = (J_1 + R) / p_a$, with J_1 is the first invariant of the net stress tensor J_1=3(p-u_a) and R is a bonding stress;

p is the mean pressure: p=(σ_1+2σ_3)/3;

p_a is a constant atmospheric pressure;

γ and β are ultimate parameters;

$\bar{S}_r$ is the stress ratio with $\bar{S}_r = \sqrt{27/2}\ J_{3D} \cdot J_{2D}^{-3/2}$,

J_{3D} is the third invariant of the deviatoric stress tensor t_{ij};

α is the hardening function defined as:

$$\alpha = \frac{a_1}{\xi^{\eta_1}} \tag{3}$$

where a_1 and η_1 are the hardening parameters and ξ is the trajectory of total plastic strains given by

$$\xi = \int (d\varepsilon^p_{ij} d\varepsilon^p_{ij})^{1/2}; \tag{4}$$

n is the phase change parameter related to the state of stress at which transition from compaction to dilation occurs or at which the change in the volume vanishes.

The plastic non-associative potential function is defined as:

$$Q \equiv J^*_{2D} - \left[-\alpha_Q (J^*_1)^n + \gamma (J^*_1)^2\right](1 - \beta\ \bar{S}_r)^{-0.5} \tag{5}$$

where

$$\alpha_Q = \alpha + \kappa(\alpha_0 - \alpha) + (1 - r_v)$$

$$r_v = \xi_v / \xi$$

ξ_v is the volumetric part of ξ

α_0 is the α at the beginning of shear loading and requires a nonassociative parameter κ.

Inside the yield surface a linear elastic behavior is assumed, defined by two more parameters: the Young's modulus E and the Poisson's ratio ν.

As such the model contains 9 material parameters.

3. NUMERICAL SIMULATIONS WITH THE MODIFIED δ_1-MODEL

Experimental results obtained on a sandy silt (Laloui *et al.*, 1997) are used to analyze the performance of the modified δ_1-model. Figure 2 shows stress-strain curves obtained from triaxial compression tests on saturated samples and from the modified δ_1-model (material parameter see Table 1). τ_{oct} is defined as:

$$\tau_{oct} = \frac{\sqrt{2}}{3} q \tag{6}$$

with $q = \sigma_1 - \sigma_3$ the deviatoric stress.

ε_1, $\varepsilon_{2,3}$ represent the strain values in the principal directions 1 and 2 or 3; ε_v is the volumetric strain.

Table 1: Material parameters (saturated state)

Parameter	Symbol	Value
Young Modulus	E	145000 kPa
Poisson's ratio	ν	0.4
Shape of F	β	0.58
Ultimate parameter	γ	0.08
Bonding stress	R	0 kPa
Hardening parameter	a_1	0.005
Hardening parameter	η_1	0.36
Phase change parameter	n	2.45
Non-assoc. parameter	κ	0.5

To simulate the unsaturated tests, the following

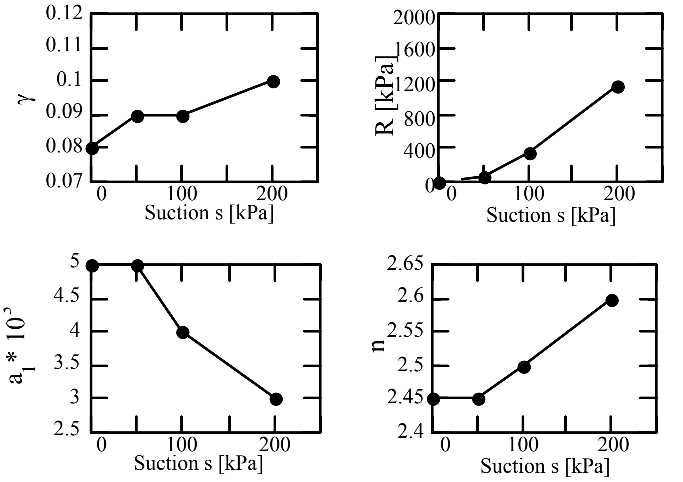

Figure 3: Experimental determination of γ, R, n and a_1 versus suction s

four parameters are taken as non linear functions of the suction: γ, R, a_1 and n. The corresponding evolution functions are determined experimentally (Figure3).

The experimental results show that the bonding stress R increases with the suction (what corresponds in more conventional terms to an increase of the cohesion). The parameter γ (which defines the slope of the ultimate envelope) only slightly increases with increasing suction while the

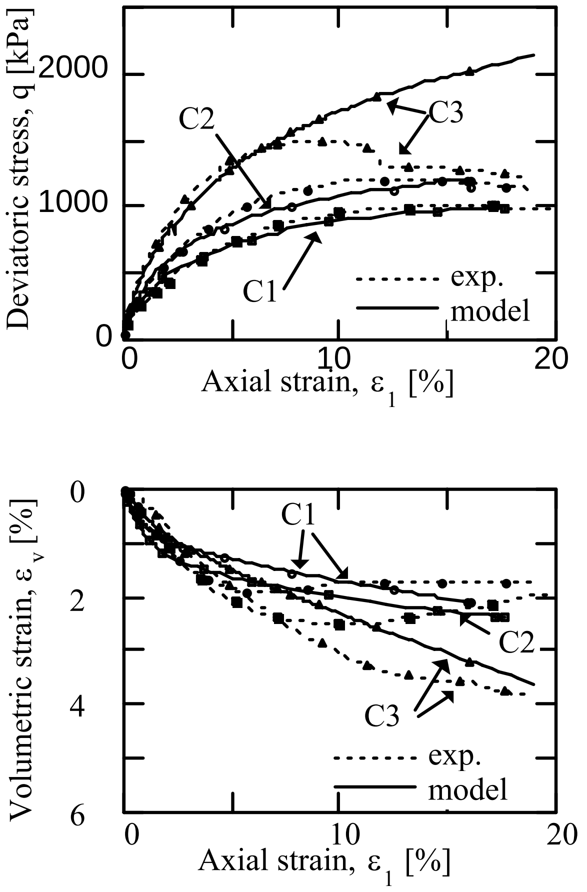

Figure 4: Simulations with the modified δ_1-model

hardening parameter a_1 decreases. This denotes a stiffening of the soil with increasing suction. The phase change parameter n seems also to be slightly affected by the suction. The parameters β, η_1, κ, E and ν are assumed for the time being to be independent of the suction.

Three drained triaxial compression tests at the same initial net mean pressure $p-u_a$=400 kPa are simulated with the modified δ_1-model, namely:

- C1, saturated case:σ_3=400 kPa, s=0kPa
- C2, small suction: σ_3=450 kPa, s=50kPa
- C3, high suction: σ_3=600 kPa, s=200kPa

Two other tests will be used later, namely
- C4, average suction: σ_3=600 kPa, s=100kPa
- C5, average suction: σ_3=400 kPa, s=100kPa

The comparison between experiments and computation for cases C1 to C3 are presented in Figure 4. The saturated case C1 corresponds to a calibration path. The numerical simulation of the test C2 (small suction, s=50 kPa) is close to the experimental results. In the case C3 (high suction) the experimental stress-strain behavior shows an increase of strength followed by a loss of resistance like a brittle failure; this effect appears more pronounced for higher suctions. Due to its intrinsic feature of continuous yielding, the modified δ_1-model is not able to predict such behavior. A possible approach is to extend the DSC theory to partially saturated soils, as presented below.

4. NEW APPROACH BASED ON THE DSC

4.1 Introduction

The use of a general concept in the framework of the disturbed state concept (DSC) (Desai, 1995) is proposed to model the particular behavior of unsaturated soils, namely the loss of strength in the stress-strain relationship for low degrees of saturation and low net mean pressures in the post-peak phase.

The DSC is based on the idea that a deforming material element can be treated as a mixture of two constituent parts in the relative intact (RI) and fully adjusted (FA) states, referred to as reference state. During external loading, the material experiences internal changes in its microstructure due to a self-adjustment process and, as a consequence, the initial RI state transforms continuously to the FA state. The observed stress $\boldsymbol{\sigma}^a_{ij}$ is defined as:

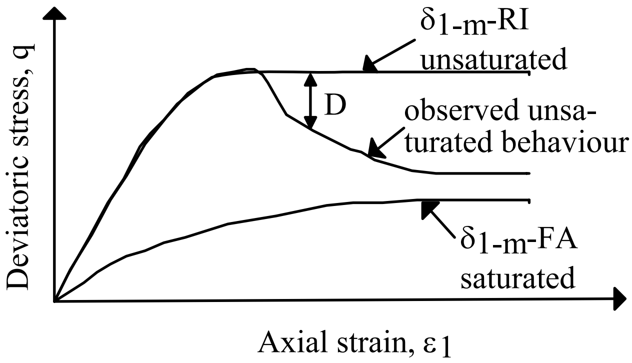

Figure 5: DSC concept for unsaturated soil

$$\sigma^a_{ij} = (1 - D)\sigma^i_{ij} + D\,\sigma^c_{ij} \tag{7}$$

where σ^i_{ij} is the RI stress, σ^c_{ij} the FA stress and D the disturbance function ($0 \le D \le 1$).

As a first approach the FA state is considered as the stress state with no disturbance (D=1). This case represents here the saturated state. The RI state is obtained with the modified δ_1-model presented in the previous section. Figure 5 shows those different states.

The disturbance function D is expressed as a scalar, which represents the microstructural changes leading to microcraking, damage and softening. It is proposed as a first approximation to express D as a function of the deviatoric stress invariant.

The disturbance function is then:

$$D = \frac{\sqrt{J_{2D}}^{\,i} - \sqrt{J_{2D}}^{\,a}}{\sqrt{J_{2D}}^{\,i} - \sqrt{J_{2D}}^{\,c}} \tag{8}$$

with the exponent "i" corresponding to the RI-state, "a" corresponding to the observed feature and "c" corresponding to the FA-state.

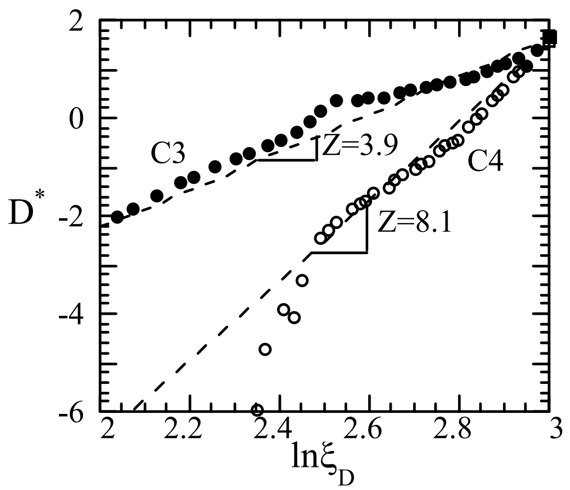

Figure 6: Determination of the parameters A and Z

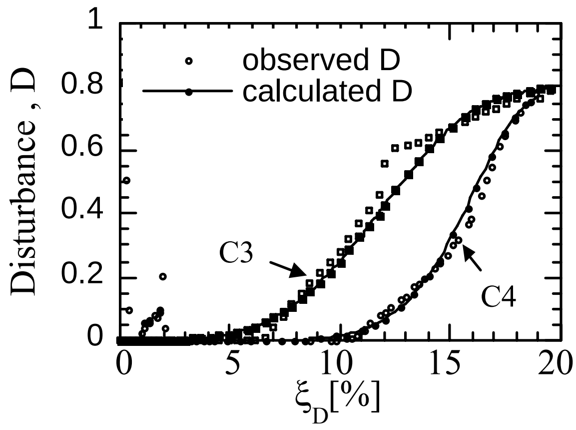

Figure 7: Comparison between the observed and calculated disturbance

The disturbance is assumed to be a function of the trajectory of the deviatoric plastic strains ξ_D and the suction s. It is expressed as:

$$D = D_u(1 - e^{-A\xi_D^Z}) \tag{9}$$

where D_u is the ultimate value of D (a material parameter), which may be assumed to be independent of the suction;

A and Z are material parameters function of the suction (see later Eq. 11);

The trajectory of the deviatoric plastic strains is defined as:

$$\xi_D = \int \left(dE^p_{ij} \cdot dE^p_{ij} \right)^{1/2} \tag{10}$$

where $dE^p_{ij} = d\varepsilon^p_{ij} - \frac{1}{3} d\varepsilon^p_v\ \delta_{ij}$

with $d\varepsilon^p_{ij}$ the plastic strain rate and $d\varepsilon^p_v = tr\left(d\varepsilon^p_{ij}\right)$

4.2 Determination of the disturbance function D

The determination of D versus ξ_D and s is done experimentally based on two unsaturated drained triaxial compression tests showing a clear "softening" behavior:

- test at a net mean pressure $p-u_a$=400 kPa with suction s=200 kPa (C3)
- test at a net mean pressure $p-u_a$=500 kPa with suction s=100 kPa (C4)

Applying Equation 8 through the entire stress path together with the computing of the plastic strain increments (from the elasto-plastic decomposition of strain) gives the evolution of D with ξ_D at a given suction.

From Equation 9, the parameters Z and A are determined as follow:

Finally a plot of D^* vs $\ln(\xi_D)$ (see Figure 6) gives the parameter Z as the slope and the parameter A as the intercept of the regression line. The values of the

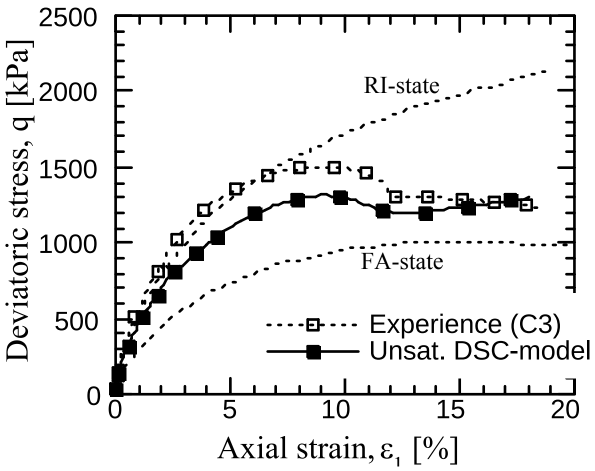

Figure 8: Comparison between the experimental result and the numerical simulation: p-ua=400 kPa, s=200 kPa (C3)

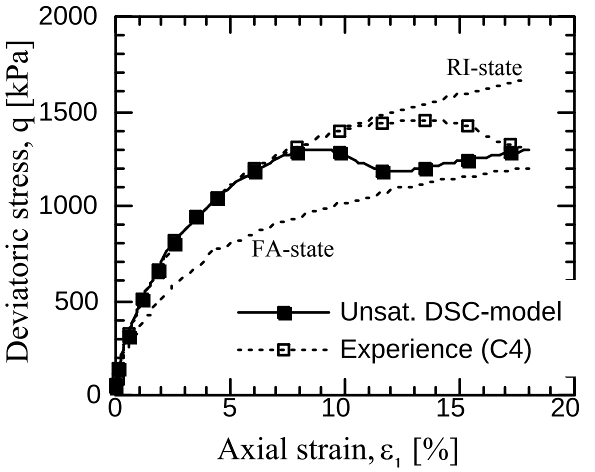

Figure 9: Comparison between the experimental result and the numerical simulation: p-ua=500 kPa, s=100 kPa (C4)

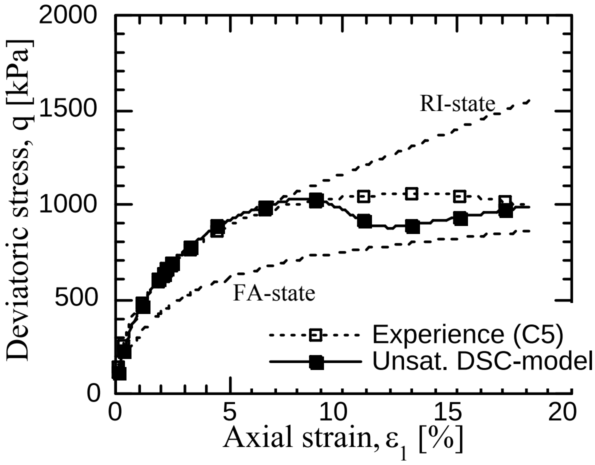

Figure 10: Comparison between the experimental result and the numerical prediction: p-ua=300 kPa, s=100 kPa (C5)

parameters are given in Table 2.

$$\ln A + Z\ln(\xi_D) = \ln\left[-\ln\left(\frac{D_u - D}{D_u}\right)\right] \equiv D^* \qquad (11)$$

Table2: Parameters of the disturbance function (Eq.6)

Suction s	Parameter Z	Parameter A	Du
100 kPa	8.1	1.4e-10	0.85
200 kPa	3.9	4.5e-5	0.85

The comparison between the observed disturbance function D (Eq. 8) and the calculated one (Eq. 9) is given on Figure 7. The correlation is very good.

4.3 Numerical simulations with the DSC concept

Figures 8 & 9 show the numerical modeling of the experimental tests used for the calibration of the material parameters (backanalysis). Using Equation 7, the unsaturated DSC gives better results than the modified δ_1-model, especially for the test C3.

Figure 10 represents the numerical prediction of a drained triaxial compression test (C5) at an initial net mean pressure of 300 kPa and a suction of 100 kPa. The comparison with the experimental results constitutes a good validation of the proposed formulation.

5. CONCLUSION

Two different models have been proposed in this paper. It is shown first that a modified δ_1-model (from the HISS elastoplatic family of constitutive model) constitutes a satisfactory model for partially saturated soil. Material parameters are simply non-linear function of the suction. However, the post peak behavior cannot be captured by this continuously yielding model. As a further improvement, the second model, unsaturated DSC (Disturbed State Concept), is capable of predicting the post peak behavior. The disturbance function can be obtained from the laboratory tests and the first results presented here are encouraging. It is believed that this unsaturated DSC has the potential to encompass most of the specific features of partially saturated soil.

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